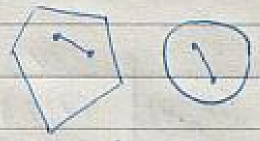
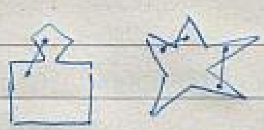
MATEMATIKA III.

P3

2005-10-20

KONVEXNÍ MNOŽINY

M $\forall a, b \in M$ obsahuje úsečku ab

 ale ne: 

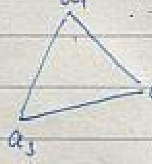
a, b ab: $a + \langle b - a \rangle$ $c = a + \lambda(b-a)$ úsečka $\lambda \in \langle 0, 1 \rangle$

Formální zápis: $c = a + \lambda b - \lambda a = (1-\lambda)a + \lambda b$

$c = \mu a + \lambda b$

$\lambda > 0 \quad \mu > 0$

$\lambda + \mu = 1$



$\lambda_1 a_1 + \dots + \lambda_k a_k$

$x_1 \geq 0; \dots; \lambda_k \geq 0 \qquad \Sigma_k = 1$

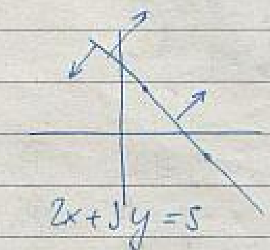
Df. $a_1; \dots; a_k \in E_n$

Každý bod $x \in E_n$

že $x = \sum_{i=1}^{k} \lambda_i a_i$; $x_i \geq 0$, $\Sigma \lambda_i = 1$ konvexní lineární kombinace bodů $a_1; \dots; a_k$

ρ nadr. $a_1 x_1 + \dots a_n x_n = b$

- uzavřená $\Sigma a_i x_i \geq b$ $\Sigma a_i x_i \leq b$
- otevřená $\Sigma a_i x_i > b$ $\Sigma a_i x_i < b$ $(0;0) = 0 \leq 5$



$2x + 3y = 5$

$(2;2) \geq 5$

•

Df. Konvexní polyedr = $\neq \emptyset$, konvexní, uzavřená a ohraničená podmnožina v E_n mající pouze konečný počet extremálních bodů

V. $M \in E_n$ $a \in M$ hraniční

$\forall$ okolí a $\exists$ body z M i mimo M

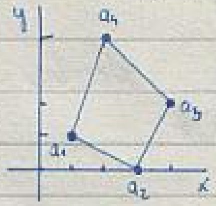
- uzavřená - obsahuje hraniční body
- otevřená - $\forall$ bod má okolí v M
- ohraničená - $\exists \varepsilon > 0 \ \forall b > 0$ je $|b - 0| < \varepsilon |b - a|$

c extremální bod M ⇔ c není vnitřní žádné úsečky v M

přímky $ax + by = c$ → lze popsat celý konvexní polyedr

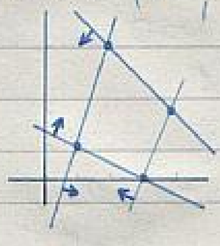
- Konvexní obal soustavou nerovností

4.19. $a_1 = [1;1]$
$a_2 = [3;0]$
$a_3 = [4;2]$
$a_4 = [2;4]$



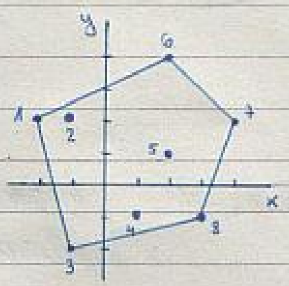
	u		n
a_1a_2	$(3-1;\ 0-1)$	$(2;-1)$	$(1;2)$
a_2a_3	$(4-3;\ 2-0)$	$(1;2)$	$(2;-1)$
a_3a_4	$(2-4;\ 4-2)$	$(2;-2)$	$(1;1)$
a_4a_1	$(2-1;\ 4-1)$	$(1;3)$	$(3;-1)$

Přímky → z n:
1,2 $x + 2y = 3$
2,3 $2x - y = 6$
3,4 $x + y = 6$
4,1 $3x - y = 2$



$x + 2y \geq 3$
$2x - y \leq 6$
$x + y \leq 6$
$3x - y \geq 2$

- $a_1 = [-2;2]$
$a_2 = [-1;2]$
$a_3 = [-1;-2]$
$a_4 = [1;-1]$
$a_5 = [2;1]$
$a_6 = [2;4]$
$a_7 = [4;2]$
$a_8 = [3;-1]$



16	$(4;2)$	$(1;-2)$	$x - 2y = -6\ \geq$
67	$(2;-2)$	$(1;1)$	$x + y = 6\ \leq$
78	$(1;3)$	$(3;-1)$	$3x - y = 10\ \leq$
83	$(4;1)$	$(1;-4)$	$x - 4y = 7\ \leq$
31	$(1;-4)$	$(4;1)$	$4x + y = -6\ \geq$

- 4.21
$2x - y \geq 2$
$x + y \geq 7$
$x + 4y \geq 10$
$x - 2y \leq 10$
$2x + 5y \leq 38$

$P_{12} = [3;4]$ $P_{23} = [8;-1]$
$P_{13} = [2;2]$ $P_{25} = [-1;8]$
$P_{14} = [-2;-6]$ $P_{34} = [10;0]$
$P_{15} = [4;6]$ $P_{35} = [34;\boxed{2}]$
$P_{23} = [6;1]$ $P_{45} = [14;\boxed{2}]$

① $P_{1..}$
$$\left(\begin{array}{cc|c} 2 & -1 & 2 \\ 1 & 1 & 7 \\ 1 & 4 & 10 \\ 1 & -2 & 10 \\ 2 & 5 & 38 \end{array}\right) \sim \left(\begin{array}{cc|c} 2 & -1 & 2 \\ 0 & -3 & -12 \\ 0 & -9 & -18 \\ 0 & 3 & -18 \\ 0 & 6 & 36 \end{array}\right)$$
z 1. rovnice: $x = \frac{y+2}{2}$

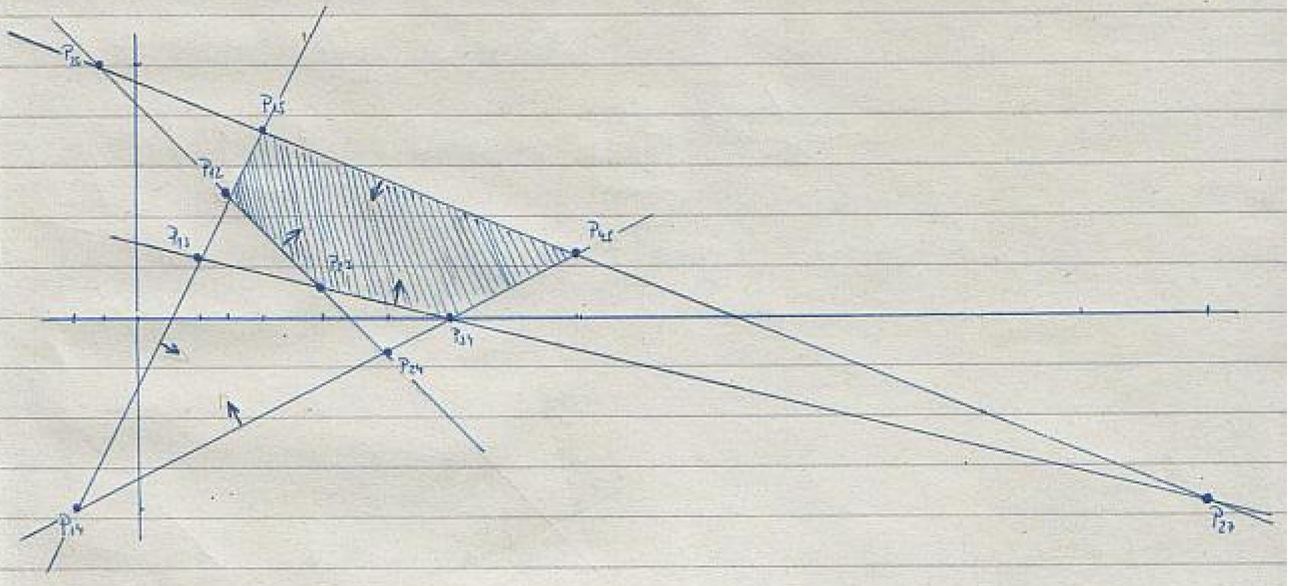
② P_2.
$$\left(\begin{array}{cc|c} 1 & 1 & 7 \\ 0 & 3 & 3 \\ 0 & -3 & 3 \\ 0 & 3 & 24 \end{array}\right) \quad 3y = 3,\ y = 1$$
z 2. rovnice: $x = 7 - y$

③
$$\left(\begin{array}{cc|c} 1 & 4 & 10 \\ 0 & -6 & 0 \\ 0 & -3 & -18 \end{array}\right) \quad -3y = -18,\ y = \boxed{6}$$
z 3. rovnice: $x = 10 - 4y$

④ $\left(\begin{array}{cc|c} 1 & -2 & 10 \\ 0 & 9 & 18 \end{array}\right) \rightarrow 9y=18$

$y=2$

ze 4. rovnice: $x=10+2y$



MATEMATIKA III.

C3

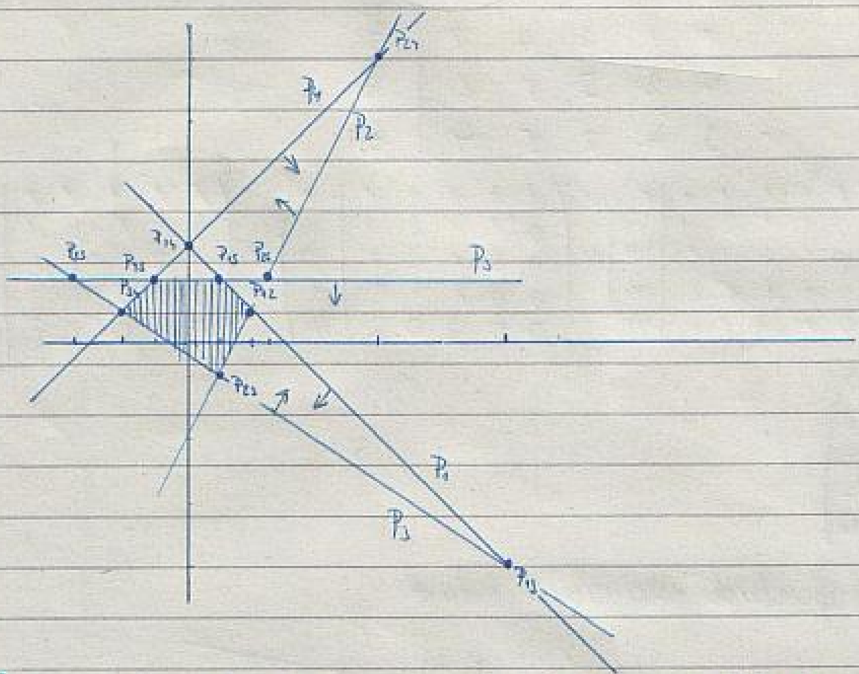
2005-10-20

KONVEXNÍ MNOŽINY

- $x+y\le 3$ $\quad P_{12}=[2;1]$ $\quad P_{24}=[6;9]$
  $2x-y\le 3$ $\quad P_{13}=[10;-7]$ $\quad P_{25}=[\frac{5}{2};2]$
  $2x+3y\ge -1$ $\quad P_{14}=[0;3]$ $\quad P_{34}=[-2;1]$
  $x-y\ge -3$ $\quad P_{15}=[1;2]$ $\quad P_{35}=[-\frac{7}{2};2]$
  $y\le 2$ $\quad P_{23}=[1;-1]$ $\quad P_{45}=[-1;2]$

① $\left(\begin{array}{cc|c} 1 & 1 & 3 \\ 2 & -1 & 3 \\ 2 & 3 & -1 \\ 1 & -1 & -3 \\ 0 & 1 & 2 \end{array}\right) \sim \left(\begin{array}{cc|c} 1 & 1 & 3 \\ 0 & -3 & -3 \\ 0 & 1 & -7 \\ 0 & -2 & -6 \\ 0 & 1 & 2 \end{array}\right)$ ② $\left(\begin{array}{cc|c} 2 & -1 & 3 \\ 0 & 4 & -4 \\ 0 & 1 & 9 \\ 0 & 1 & 3 \end{array}\right)$ ③ $\left(\begin{array}{cc|c} 2 & 3 & -1 \\ 0 & 5 & 5 \\ 0 & 1 & 2 \end{array}\right)$ ④ $\left(\begin{array}{cc|c} 1 & -1 & -3 \\ 0 & & \end{array}\right)$

$x=3-y$ $\qquad y=\frac{3+y}{2}$ $\qquad y=\frac{-1-3y}{2}$



Df. $a_0; a_1; \dots; a_k \in E_n$ jsou lineárně nezávislé (LN) jestliže vektory $a_1-a_0; \dots; a_k-a_0$ jsou lineárně nezávislé (LN)

- $a_0=[2;1;4;3]$ $\quad a_3=[4;0;5;5]$
  $a_1=[3;2;3;4]$ $\quad a_4=[1;2;6;4]$
  $a_2=[3;3;5;2]$

$$\begin{array}{c} a_1-a_0 \\ a_2-a_0 \\ a_3-a_0 \\ a_4-a_0 \end{array}\begin{pmatrix} 1 & 1 & -1 & 1 \\ 1 & 2 & 1 & -1 \\ 2 & -1 & 1 & 2 \\ -1 & 1 & 2 & 1 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & -1 & 1 \\ 0 & 1 & 2 & -2 \\ 0 & -3 & 3 & 0 \\ 0 & 2 & 1 & 2 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & -1 & 1 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 3 & -2 \\ \cancel{0} & \cancel{0} & \cancel{3} & \cancel{2} \end{pmatrix}$$

$$0 \quad 1 \quad -1 \quad 0 \qquad\qquad 0 \quad 0 \quad 0 \quad 1$$

n-rozměrný simplex v E_n

$a_0; \dots; a_n$ lin. nezávislé

$$\varphi^n(a_0; \dots; a_n) = \left\{ x = \sum_{i=0}^{n} \lambda_i a_i \;\middle|\; \sum_{i=0}^{n} \lambda_i = 1,\ \lambda_i \geq 0 \right\}$$

$(\lambda_0; \lambda_1; \dots; \lambda_n)$ – **barycentrické souřadnice** bodu x

- **Barycentrické souřadnice** $\quad \lambda_0 a_0 + \lambda_1 a_1 + \lambda_2 a_2 + \lambda_3 a_3 + \lambda_4 a_4 = 0$

  $b = \left[\frac{16}{7}; \frac{11}{7}; \frac{33}{7}; \frac{25}{7}\right]$

  vektory viz předchozí příklad

$$\begin{array}{ccccc|c} \lambda_0 & \lambda_1 & \lambda_2 & \lambda_3 & \lambda_4 & \\ 2 & 3 & 3 & 4 & 1 & \frac{16}{7} \\ 1 & 2 & 3 & 0 & 2 & \frac{11}{7} \\ 4 & 3 & 5 & 5 & 6 & \frac{23}{7} \\ 3 & 4 & 2 & 5 & 4 & \frac{25}{7} \end{array}$$

$$\left(\begin{array}{ccccc|c} 2 & 3 & 3 & 4 & 1 & 16/7 \\ 1 & 2 & 3 & 0 & 2 & 11/7 \\ 4 & 3 & 5 & 5 & 6 & 23/7 \\ 3 & 4 & 2 & 5 & 4 & 25/7 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{array}\right) \sim \left(\begin{array}{ccccc|c} 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 2 & -1 & 2/7 \\ 0 & 1 & 2 & -1 & 1 & 4/7 \\ 0 & -1 & 1 & 1 & 2 & 5/7 \\ 0 & 1 & -1 & 2 & 1 & 4/7 \end{array}\right) \sim \left(\begin{array}{ccccc|c} 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 2 & -1 & 2/7 \\ 0 & 0 & 1 & -3 & 2 & 2/7 \\ 0 & 0 & 2 & 1 & 1 & 1 \\ 0 & 0 & -2 & 0 & 2 & 2/7 \end{array}\right) \sim \left(\begin{array}{ccccc|c} 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 2 & -1 & 2/7 \\ 0 & 0 & 1 & -3 & 2 & 2/7 \\ 0 & 0 & 0 & 9 & -3 & 3/7 \\ 0 & 0 & 0 & -6 & 6 & 6/7 \end{array}\right)$$

$$\sim \left(\begin{array}{ccccc|c} 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 2 & -1 & 2/7 \\ 0 & 0 & 1 & -3 & 2 & 2/7 \\ 0 & 0 & 0 & -1 & 1 & 1/7 \\ 0 & 0 & 0 & 0 & 6 & 12/7 \end{array}\right)$$

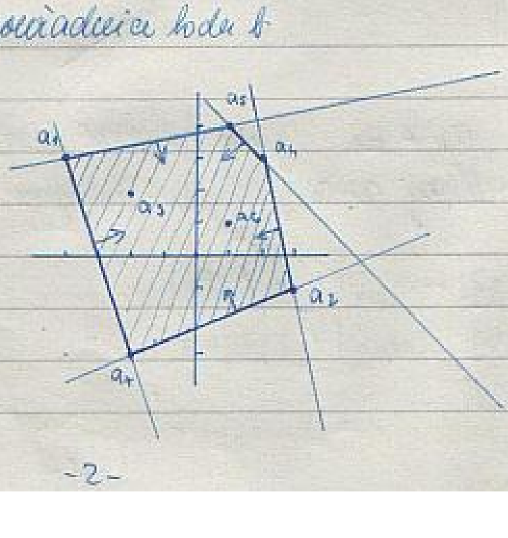
$\left(\frac{2}{7}; \frac{1}{7}; \frac{1}{7}; \frac{1}{7}; \frac{2}{7}\right)$ barycentrické souřadnice bodu b

- $a_1 = [-4; 3] \qquad a_5 = [1; 4]$

  $a_2 = [3; -1] \qquad a_6 = [1; 1]$

  $a_3 = [-2; 2] \qquad a_7 = [-2; -3]$

  $a_4 = [2; 3]$



15	$(-5;-1)$	$(1;-5)$	$x-5y=-19$	$\geq$
54	$(1;-1)$	$(1;1)$	$x+y=5$	$\leq$
42	$(1;-4)$	$(4;1)$	$4x+y=11$	$\leq$
27	$(5;2)$	$(2;-5)$	$2x-5y=11$	$\leq$
71	$(-2;6)$	$(3;1)$	$3x+y=-9$	$\geq$