MATEMATIKA III. C4

2004-10-22

- $[2;1;1]+\langle(4;1;3);(1;1;1)\rangle$

$$\begin{pmatrix}4&1&3\\1&1&1\end{pmatrix}\sim\begin{pmatrix}1&1&1\\4&1&3\end{pmatrix}\overset{\cdot(-4)}{\sim}\begin{pmatrix}1&1&1\\0&-3&-1\end{pmatrix}$$

$$\begin{pmatrix}1&1&1\\0&-3&-1\end{pmatrix}$$

$x\quad y\quad 1\qquad \vec{n}=(2;1;-3)$

$2x+y-3z-2=0$

$A=[1;3;2]$

$2+3-6-2=-3\neq 0 \qquad A\in\rho$

$r=\dfrac{3}{\sqrt{14}}$

* $P[x_0;y_0]$

$ax+by+c=0 \qquad r=\dfrac{|ax_0+by_0+c|}{\sqrt{a^2+b^2}}$

* $P[x_1^0;x_2^0;\ldots x_n^0]$

$a_1x_1+a_2x_2+\ldots+a_nx_n+d=0 \qquad r=\dfrac{|a_1x_1^0+a_2x_2^0+\ldots+a_nx_n^0+d|}{|\vec{n}|}$

- E_3: $[4;1;-2]+\langle(4;2;-1);(1;4;5)\rangle$

$C=[5;-5;5]$ Normála roviny v bodě C + průsečík d.

$$\begin{pmatrix}4&2&-1\\1&4&5\end{pmatrix}\sim\begin{pmatrix}1&4&5\\4&2&-1\end{pmatrix}\overset{\cdot(-4)}{\sim}\begin{pmatrix}1&4&5\\0&-14&-21\end{pmatrix}$$

$$\begin{pmatrix}1&4&5\\0&-14&-21\end{pmatrix}$$

$x\quad y\quad 1\qquad \vec{n}=(2;-3;2)\qquad p_n=[5;-5;5]+\langle(2;-3;2)\rangle$

$2x-3y+2z+d=0$

$2\cdot4-3\cdot1+2\cdot(-2)+d=0 \qquad x=5+2t$

$8-3-4+d=0 \qquad y=-5-3t$

$1+d=0 \qquad z=5+2t$

$d=-1$

rovina: $2x-3y+2z-1=0$

- $\rho = [1;1;1;1] + \langle (5;4;-22;-11);(5;-29;11;22) \rangle$

$$\begin{pmatrix} 5 & 4 & -22 & -11 \\ 5 & -29 & 11 & 22 \end{pmatrix}^{(-1)} \sim \begin{pmatrix} 5 & 4 & -22 & -11 \\ 0 & -33 & 33 & 33 \end{pmatrix} \sim \begin{pmatrix} 5 & 4 & -22 & -11 \\ 0 & -1 & 1 & 1 \end{pmatrix}$$

$$\begin{matrix} x & y & 1 & 0 \\ x & y & 0 & 1 \end{matrix}$$

$-y+1=0 \quad y=1 \qquad 5x+4-22=0 \quad 5x=18 \quad x=\frac{18}{5}$

$-y+1=0 \quad y=1 \qquad 5x+4-11=0 \quad 5x=7 \quad x=\frac{7}{5}$

$\vec{n} = (18;5;5;0)$

$\vec{n} = (7;5;0;5)$

$18x+5y+5z-d=0 \rightarrow 18x+5y+5z-28=0$

$7x+5y+5z+d=0 \rightarrow 7x+5y+5z-17=0$

- $\rho = [\underset{x}{1};\underset{y}{-1};\underset{z}{1};\underset{t}{0};\underset{u}{2}] + \langle (2;-3;-1;-2;2);(1;-1;2;-1;3) \rangle$

$$\begin{pmatrix} 2 & -3 & -1 & -2 & 2 \\ 1 & -1 & 2 & -1 & 3 \end{pmatrix} \sim \begin{pmatrix} 1 & -1 & 2 & -1 & 3 \\ 2 & -3 & -1 & -2 & 2 \end{pmatrix}^{(-2)} \sim \begin{pmatrix} 1 & -1 & 2 & -1 & 3 \\ 0 & -1 & -5 & 0 & -4 \end{pmatrix}$$

$$\begin{pmatrix} 1 & -1 & 2 & -1 & 3 \\ 0 & -1 & -5 & 0 & -4 \end{pmatrix}$$

$$\begin{matrix} x & y & z & 1 & 0 \\ x & y & z & 0 & 1 \end{matrix}$$

$(-7;-5;1;0;0) \quad \cdot(-1)$

$(5;5;-1;0;0) = \vec{n}_1$

$(1;0;0;1;0) = \vec{n}_2$

$(7;4;0;0;-1) = n_3$

$7x+5y-1z+d=0 \rightarrow 7-5-1+d=0$

$d=-1 \qquad 7x+5y-z-1=0$

$x+t+d=0 \rightarrow 1+0+d=0$

$d=-1 \qquad x+t-1=0$

$7x+4y-u+d=0 \rightarrow 7-4-2+d=0$

$d=-1 \qquad 7x+4y-u-1=0$

$x-2y+3z+2t=4$

$2x-y+5z+3t=9$

$$\left(\begin{array}{cccc|c} 1 & -2 & 3 & 2 & 4 \\ 2 & -1 & 5 & 3 & 9 \end{array}\right)^{(-2)} \sim \left(\begin{array}{cccc|c} 1 & -2 & 3 & 2 & 4 \\ 0 & 3 & -1 & -1 & 1 \end{array}\right)$$

$x \quad y \quad z \quad t$

$y=t_1$

$t=t_2$

$-z=1-3y+t$

$x=7-7y+t$

$x = 7 - 7t_1 + t_2$

$y = t_1$

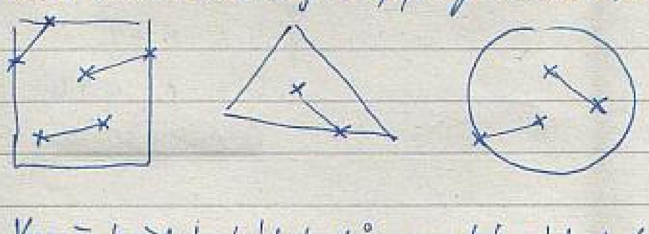
$z = -1 + 3t_1 - t_2$

$t = t_2$

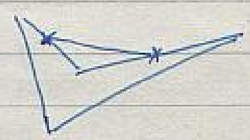
$\rho = [7; 0; -1; 0] + \langle (7; 1; 3; 0); (1; 0; -1; 1) \rangle$

KONVEXNÍ MNOŽINA:

Vezmu-li libovolné dva body a spojím je úsečkou => úsečka v množině zůstává.



ALE:



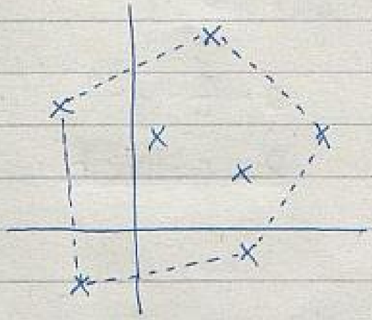
Konečný počet krajních bodů -> extrémní body (vrcholy)

KONVEXNÍ POLYEDR:

Neprázdná konvexní množina; ohraničená, uzavřená, s konečným počtem krajních bodů.

LINEÁRNÍ KONVEXNÍ OBAL:

Nejmenší konvexní množina, která obsahuje všechny body.



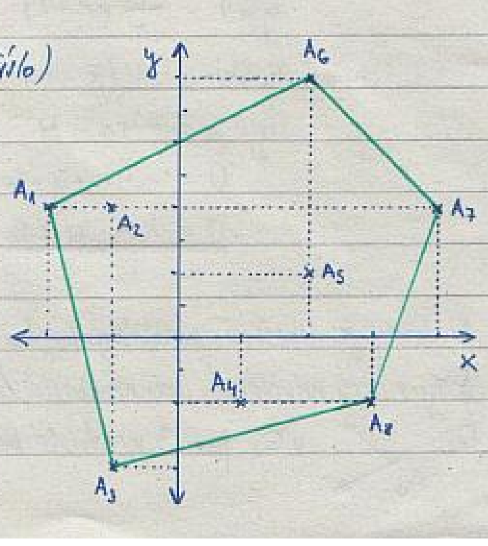
Všechny body uvnitř lze vyjádřit pomocí lineární konvexní kombinace vrcholů

$c_1A_1 + c_2A_2 + \ldots + c_5A_5 \qquad c_1 + \ldots c_5 = 1 \qquad c_i \geq 0$

A - body

c - koeficient (reálné číslo)

- $A_1 = [-2; 2]$ $\quad A_5 = [2; 1]$
  $A_2 = [-1; 2]$ $\quad A_6 = [2; 4]$
  $A_3 = [-1; -2]$ $\quad A_7 = [4; 2]$
  $A_4 = [1; -1]$ $\quad A_8 = [3; -1]$



$A = [x_1; y_1]$
$B = [x_2; y_2]$

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$A_1 = [-2; 2]$
$A_6 = [2; 4]$

$$y - 2 = \frac{4-2}{2-(-2)}(x-(-2))$$
$$y - 2 = \frac{2}{4}(x+2)$$
$$y - 2 = \frac{1}{2}(x+2)$$
$$y - 2 = \frac{x}{2} + 1$$
$$0 = \frac{x}{2} - y + 3$$
$$\underline{\underline{0 = x - 2y + 6}}$$

$A_6 = [2; 4]$
$A_7 = [4; 2]$

$$y - 4 = \frac{2-4}{4-2}(x-2)$$
$$y - 4 = \frac{-2}{2}(x-2)$$
$$y - 4 = -1(x-2)$$
$$y - 4 = -x + 2$$
$$0 = -x - y + 6$$
$$\underline{\underline{0 = x + y - 6}}$$

$A_7 = [4; 2]$
$A_8 = [3; -1]$

$$y - 2 = \frac{-1-2}{3-4}(x-4)$$
$$y - 2 = \frac{-3}{-1}(x-4)$$
$$y - 2 = 3(x-4)$$
$$y - 2 = 3x - 12$$
$$\underline{0 = 3x - y - 10}$$

$A_8 = [3; -1]$
$A_3 = [-1; -2]$

$$y - (-1) = \frac{-2-(-1)}{-1-3}(x-3)$$
$$y + 1 = \frac{-2+1}{-4}(x-3)$$
$$y + 1 = \frac{-1}{-4}(x-3)$$
$$y + 1 = \frac{1}{4}(x-3)$$
$$y + 1 = \frac{x}{4} - \frac{3}{4}$$
$$0 = \frac{x}{4} - y - \frac{7}{4}$$
$$\underline{\underline{0 = x - 4y - 7}}$$

$A_3 = [-1; -2]$
$A_1 = [-2; 2]$

$$y - (-2) = \frac{2-(-2)}{-2-(-1)}(x-(-1))$$
$$y + 2 = \frac{2+2}{-2+1}(x+1)$$
$$y + 2 = \frac{4}{-1}(x+1)$$
$$y + 2 = -4(x+1)$$
$$y + 2 = -4x - 4$$
$$0 = -4x - y - 6$$
$$\underline{\underline{0 = 4x + y + 6}}$$

$$x - 2y + 6 \geq 0$$
$$x + y - 6 \leq 0$$
$$3x - y - 10 \leq 0$$
$$4x - y + 6 \geq 0$$
$$x - 4y - 7 \leq 0$$

* N-bodů je lineárně nezávislých, když body z nich vytvořené jsou nezávislé (i vektory)

E_2: 

v E_2 jsou 3 nezávislé body

SIMPLEX: